

# Algebraic, analytic and geometric structures emerging from quantum field theory

4-16 March 2024, Chengdu, China

Organisers:      Bin Zhang (Chengdu)      Li Guo (Rutgers)      Sylvie Paycha (Potsdam)

ALGEBRAIC, ANALYTIC AND GEOMETRIC STRUCTURES  
EMERGING FROM QUANTUM FIELD THEORY

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Introductory courses: March 1–3 (West 303, Math Building)			
Times	Friday	Saturday	Sunday
8:30–9:20	Guo	Guo	Guo
9:30–10:20	Guo	Guo	Guo
10:30–11:20	Paycha	Paycha	Paycha
11:30–14:00	Lunch break		
14:00–14:50	Paycha	Paycha	Paycha
15:00–15:50	Zhang	Zhang	Zhang
15:50–16:30	Question break		
16:30–17:20	Zhang	Zhang	Zhang

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## Abstracts: Introductory Courses (March 1–3)

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### Li Guo

Rutgers University

TITLE: **Algebraic and combinatorial prerequisites**

- Hopf algebras
- Algebraic combinatorics
- Non-associative algebras

### Sylvie Paycha

Universität Potsdam

TITLE: **Geometric and analytic prerequisites**

- Distributions, Wave front sets
- Pseudodifferential operators, Fourier integral operators
- Index theory

### Bin Zhang

Sichuan University, Chengdu

TITLE: **Differential geometric and algebraico-geometric prerequisites**

- Vector bundle, principal bundle
- Connection, curvature, characteristic classes

**Mini-courses: March 4–8 (West 303, Math Building)**

<b>Times</b>	<b>Monday</b>	<b>Tuesday</b>	<b>Wednesday</b>	<b>Thursday</b>	<b>Friday</b>
8:30–9:20	<b>Bellingeri</b>	<b>Cren</b>	<b>Fereira</b>	<b>Vargas</b>	<b>Zanello</b>
9:30–10:20	<b>Cren</b>	<b>Fereira</b>	<b>Vargas</b>	<b>Zanello</b>	<b>Bellingeri</b>
10:20–10:40	<b>Break</b>				
10:40–11:30	<b>Vargas</b>	<b>Fereira</b>	<b>Zanello</b>	<b>Bellingeri</b>	<b>Cren</b>
11:30–14:00	<b>Lunch break</b>				
14:00–14:50	<b>Vargas</b>	<b>Zanello</b>	<b>Bellingeri</b>	<b>Cren</b>	<b>Fereira</b>
15:00–15:50	<b>Zanello</b>	<b>Bellingeri</b>	<b>Cren</b>	<b>Fereira</b>	<b>Vargas</b>
15:50–16:30	<b>Break</b>				
16:30–17:30	<b>Jiasheng Lin</b>	<b>Guodong Zhou</b>	<b>Yuanyuan Zhang</b>	<b>Jean-David Jacques</b>	<b>Honglian Zhang</b>

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## Abstracts: Mini-courses (March 4-8)

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### Carlo Bellingeri

Université de Lorraine, Nancy

TITLE: **Hopf algebras and randomness**

In this series of lectures, I will give an introduction to the theory of Hopf algebras and their applications to some new and old problems in stochastics. I'll start from the usual setting of graded connected Hopf algebras and explain how these objects represent the abstract framework to describe features of random objects such as random variables, stochastic processes and random fields. A fundamental example to understand these constructions is given by the well-known Connes-Kreimer Hopf algebra. I will review its main properties and show how the explicit knowledge of its primitive elements allows the construction of a consistent pathwise stochastic calculus on general stochastic processes. Some content of this mini-course will be based on an ongoing work with Emilio Rossi Ferrucci and Nikolas Tapia.

### Clément Cren

Georg-August Universität, Göttingen, Germany

TITLE: **Pseudodifferential calculus and deformation groupoids**

In recent years the theory of deformation groupoids has given new insights on pseudodifferential calculus. We know since the early work of Connes the link between the tangent groupoid and the analytical index of such operators. More recent work have deepened this bond by showing how one could define pseudodifferential operators in a completely intrinsic fashion using the tangent groupoid (even without using distributions).

In this series of lectures we will define the tangent groupoid of a manifold and show how to construct pseudodifferential operators from it. No prior knowledge on pseudodifferential operators nor groupoids is necessary. We will compare the two different approaches to pseudodifferential calculus from the tangent groupoid (the one of Debord-Skandalis on one hand and van Erp-Yuncken on the other). Depending on time we might see how this approach generalizes to other pseudodifferential calculi, more suited for a sub-riemannian setting for instance, or explore the link between the deformations of the tangent groupoid and Getzler's rescaling technique.

### Alexandre Ferreira

Universidade Federal do Espírito Santo (Vitoria, Brazil)

TITLE: **Deriving quantum dynamics from maps between manifolds**

In this minicourse I will introduce a generalized dynamics, given through maps between manifolds, the source space being the configuration space and the target representing the internal structure of the field. Under some simple assumptions, we can derive from there the Schroedinger equation. The latter yields interesting geometrical interpretations for some of its uncanny properties, such as non-locality. The minicourse will start with a thorough

review of manifold mapping theory and its applications in physics. Subsequently, I will introduce the Schroedinger map, establishing the foundation for the generalized dynamics, and show how classical and quantum mechanics emerge with some possible generalizations.

### Yannic Vargas

Technische Universität, Graz, Austria

**TITLE: Introduction to combinatorial species by means of algebraic combinatorics, geometry and physics**

This short course is intended to provide an introduction to the theory of combinatorial species, with a focus on various (new) applications that have emerged from this theory over the last two decades. In 1981, André Joyal introduced this notion as “a combinatorial theory of formal power series,” and since then, the field has garnered significant attention as a formal foundation for reinterpreting combinatorial aspects of various problems in Hopf algebras, hyperplane arrangements and quantum field theory.

### Fabrizio Zanella

Universität Potsdam, Germany

**TITLE: Introduction to perturbative algebraic quantum field theory on Minkowski spacetime**

In these lectures we present the algebraic approach to Quantum Field Theory in its simplest framework, namely, on flat Lorentzian spacetime. We first introduce observables as elements of an abstract algebra, prior to any representation on some Hilbert space. We then outline the appropriate analytical framework, essentially based on microlocal analysis techniques, which allows to interpret a quantum field theory as a deformation quantization of its classical counterpart. Finally, with these notions at hand, we approach the interaction picture from the point of view of the so-called causal perturbation theory and, if time permits, we formulate the renormalization problem in a mathematically rigorous way.

### Workshop: March 11–15 (Even Hotel)

<b>Times</b>	<b>Monday</b>	<b>Tuesday</b>	<b>Wednesday</b>	<b>Thursday</b>	<b>Friday</b>
<i>Chair</i>	<i>Sylvie Paycha</i>	<i>Alessandra Frabetti</i>	<i>Fay Dowker</i>	<i>Carlo Bellingeri</i>	<i>Jorge Zanelli</i>
8:30–9:20	<b>David Sauzin</b>	<b>Si Li</b>	<b>Jorge Zanelli</b>	<b>Carlos Amendola</b>	<b>Cuibo Jiang</b>
9:30–10:20	<b>Yong Li</b>	<b>Alberto Richtsfeld</b>	<b>Xin Wang</b>	<b>Honglei Lang</b>	<b>Yangyong Hong</b>
10:20–10:50	<b>Break</b>				
10:50–11:40	<b>Sumati Surya</b>	<b>Hang Wang</b>	<b>Xiaomeng Xu</b>	<b>Yunhe Sheng</b>	<b>Alexandre Ferreira</b>
12:00–14:00	<b>Lunch</b>				
<i>Chair</i>	<i>Bin Zhang</i>	<i>David Sauzin</i>		<i>Reiko Toriumi</i>	<i>Sumati Surya</i>
14:00–14:50	<b>Fay Dowker</b>	<b>Clément Cren</b>		<b>Harprit Singh</b>	<b>Reiko Toriumi</b>
15:00–15:50	<b>Fabrizio Zanello</b>	<b>Raphaël Ponge</b>		<b>Martin Peev</b>	<b>Jiasheng Lin</b>
15:50–16:20	<b>Break</b>				
16:20–17:10	<b>Sarah-Jean Meyer</b>	<b>Gihyun Lee</b>		<b>Pierre Clavier</b>	<b>Naihong Hu</b>
17:20–18:10	<b>Carlo Bellingeri</b>	<b>Fang Li</b>		<b>Yannic Vargas</b>	<b>Alessandra Frabetti</b>
<b>18:30</b>		<b>Conference dinner</b>		<b>19:30: Film: Words of women...</b>	

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## Abstracts: Workshop (March 11–15)

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### Carlos Amendola

Technische Universität Berlin, Germany

TITLE: **From path signatures to algebraic varieties**

The signature of a parametric curve is a sequence of tensors whose entries are iterated integrals. This construction is central to the theory of rough paths in stochastic analysis. We examine such path signatures through the lens of algebraic geometry by introducing varieties of signature tensors for both deterministic paths and random paths. In this way, we present a bridge between applied algebraic geometry and stochastic analysis where rough paths are encoded in signature tensors, and whose varieties offer a concise representation of geometric data seen in applications.

### Carlo Bellingeri

Université de Nancy 1, France

TITLE: **The Euler-Maclaurin formula and generalised iterated integrals**

Considered one of the key identities in classical analysis, the Euler-McLaurin formula is one of the standard tools for relating sums and integrals, with remarkable applications in many areas of mathematics, although it is little used in stochastic analysis. In this talk, we will show how, by introducing new variants of the iterated integrals of a path and a simple variational problem, we can generalise this identity in the context of Riemann Stieltjes integration. Joint work with Sylvie Paycha (Potsdam) and Peter Friz ( TU Berlin and WIAS)

### Pierre Clavier

Université de Haute Alsace, Mulhouse, France

TITLE: **Grafting and cografting for species**

Recall that a species is a functor from the category of sets to the category of vector spaces. An example that will serve as a common thread throughout this non-technical talk is the species of rooted trees. To a set it associates the vector space spanned by all rooted trees decorated by the set under consideration. I will describe classical operations on species and present species with products (monoid species) and further equipped with up and down operators. Following the structure of rooted trees, I will introduce grafting and cografting of species, and show that it leads to natural up and down operators with interesting algebraic properties. Finally I will show that the species of trees has a universal property in the category of monoids with up operators.

Work in collaboration with Yannic Vargas and Sylvie Paycha.

### Clément Cren

Georg-August-Universität Göttingen, Germany

**TITLE: Transversal index theory and filtered calculus**

Abstract: We know since the work Atiyah and Singer that elliptic (pseudo)differential operators are Fredholm and that their index can be computed by algebro-geometric methods. When the operator is not elliptic, interesting things can still happen. On the one hand, the symbol does not take into account a whole subbundle that forms a foliation. The operator can then be interpreted (following Atiyah, Connes, Hilsum-Skandalis...) as an elliptic operator on the space of leaves of the foliation (which in general fails to be a genuine manifold). On the other hand, the operator can only be elliptic on a certain subbundle but this subbundle generates all the whole tangent space through the Lie bracket of sections (typically a sub-laplacian in sub-riemannian geometry). Then the operator is still Fredholm but understanding this goes through a new pseudodifferential calculus. This calculus heavily relies on the filtration given by the aforementioned subbundle and the subbundles obtained by iteration of the Lie bracket of sections and has recently known huge developments (van Erp-Yuncken, Androulidakis-Mohsen-Yuncken). In this calculus the ellipticity condition is replaced by the so-called Rockland condition, involving irreducible representations of certain nilpotent groups. In this talk we will combine the two situations and consider operators that suffer both of these flaws. We set the adequate geometric context of foliated filtration, this corresponds to foliated manifolds whose space of leaves has a filtered (e.g. sub-riemannian) structure. We then define a transverse Rockland condition and proceed to construct K-theory classes from operators satisfying this condition and show the relations between these classes.

**Fay Dowker**

Imperial College, London, England

**TITLE: A diagrammatic calculus for interacting scalar QFT on a causal set**

An interacting QFT on a causal set has been independently proposed by Dable-Heath, Fewster, Rejzner and Woods and by Sorkin, Albertini and Jubb. I will show how to use the "2nd quantized" form of QFT to calculate in-in time ordered correlators, order by order in coupling, using a diagrammatic calculus. I will describe the rules for generating the diagrams, which are manifestly causal.

**Alexandre Fereira**

Federal University of Itajuba, Brazil

**TITLE: On the physics of degenerate dynamical systems**

Dynamical systems typically possess a constant-rank symplectic structure, indicating that the determinant of the Lagrangian Hessian either differs from or equals zero throughout their evolution. The latter scenario implies that certain degrees of freedom are not physical. Consequently, degenerate systems represent a generalization of standard dynamics; in these systems, the determinant has a simple zero only on a surface in phase space known as the degenerate surface. On this surface, the system can become trapped, leading to the loss of some degrees of freedom. This phenomenon, observed initially in Lovelock gravity theories for dimensions greater than 4, is a prominent characteristic of gravitation and supergravity theories. In these theories, the dynamic loss of degrees of freedom serves as a mechanism for dimensional reduction, making it a significant potential feature of UV completions of current theories. In this presentation, I will explore various properties of degenerate systems, both classical and quantum, and provide examples of field theories where this phenomenon can occur.

**Alessandra Frabetti**

Université de Lyon 1, France

**TITLE: Non-associative renormalization group**

Renormalization Hopf algebras represent the renormalization group of (perturbative) quantum field theory as a functor. In practice, this is a group of formal diffeomorphisms in the powers of the coupling constant with coefficients given by the counterterms of divergent graphs, and the "functorial representation" makes sense only for commutative coefficients and commutative Hopf algebras. However, in quantum electrodynamics and in quantum chromodynamics the coefficients of the series and the natural renormalization Hopf algebras are not commutative. In this talk I explain how we can still use them to represent a renormalization action, if we renounce to the associativity of the "group" and see it as a "loop" (a non-associative group).

### Yanyong Hong

Hangzhou Normal University, China

**TITLE: Infinite-dimensional Lie bialgebras via affinization of (multi-)Novikov bialgebras**

In this talk, I will introduce some basic facts on Novikov algebras, multi-Novikov algebras and infinite-dimensional Lie algebras. Then I will introduce the theory of (multi-) Novikov bialgebras and their connections with infinite-dimensional Lie bialgebras. This talk is based on joint works with Chengming Bai and Li Guo.

### Naihong Hu

East China Normal University, China

**TITLE: Some quantization questions related to the two-parameter quantum groups**

This is a survey talk including some recent results joint with Xu Xiao Xu, Hengyi Wang and Rushu Zhuang. One is about the isoclasses of one-parameter exotic quantum small groups at roots of unity, The second is about the descriptions of the Harish-Chandra homomorphism Theorem for two-parameter quantum groups of even rank. If the time permits, I will also talk about the 3rd one, which is about the RLL realizations of two-parameter quantum (affine) algebras of BCD types, which establishes an isomorphism between the Drinfeld new realization of the two-parameter quantum affine algebras and the RLL realization of them via RLL relations in the matrix form.

### Jean-David Jacques

Universität Potsdam, Germany

**TITLE: Post-Lie algebra of derivations and regularity structures**

Post-Lie algebra structures are a generalization of Pre-Lie algebras. They have their roots in geometry and correspond to the algebraic properties satisfied by the covariant derivative in the case of a flat and constant torsion connection. In my talk, I will provide a brief overview of the new theory of regularity structures developed by F. Otto and colleagues, and discuss how post-Lie algebra structures arise in this framework

### Cuibo Jiang

Shanghai Jiaotong University, China

**TITLE: Singular vectors and associated varieties of vertex algebras of A type**

Given the simple Lie algebras  $sl_n$  and  $k = -n + \frac{p}{q}$  such that  $p, q \in \mathbb{Z}_+$ ,  $p \geq 2$  and  $(p, q) = 1$ , let  $V^k(sl_n)$  be the associated universal vertex algebras. We will talk about recent progress on determining singulars of  $V^k(sl_n)$  and associated varieties of the simple quotients of  $V^k(sl_n)$ . In particular, we completely determine the varieties of the simple affine vertex algebras  $L_k(sl_3)$  and the simple associated affine  $W$ -algebras.

Honglai Lang

China Agricultural University, Beijing, China

TITLE: **Rota-Baxter Lie bialgebras and Rota-Baxter Poisson Lie groups**

First we introduce the notion of quadratic Rota–Baxter Lie algebras of arbitrary weight, and show that there is a one-to-one correspondence between factorizable Lie bialgebras and quadratic Rota–Baxter Lie algebras of nonzero weight. Then we introduce the notions of matched pairs, bialgebras and Manin triples of Rota–Baxter Lie algebras, and show that Rota–Baxter Lie bialgebras, Manin triples of Rota–Baxter Lie algebras and certain matched pairs of Rota–Baxter Lie algebras are equivalent. Finally, we present some results on Rota-Baxter Poisson Lie Groups. This is joint work in progress with Yunhe Sheng.

Gihyun Lee

Universiteit Gent, Belgium

TITLE: **A calculus for magnetic pseudodifferential super operators**

Abstract: The time evolution of a physical state is determined by the Liouville equation  $\frac{d\rho}{dt} = -\frac{i}{\hbar}(H\rho - \rho H)$  in quantum mechanics. Here  $\rho$  is the density operator describing a given physical state and  $H$  is the Hamiltonian of a given system. Here we can observe that the Liouville operator  $\rho \mapsto L_H\rho := -\frac{i}{\hbar}(H\rho - \rho H)$  is a super operator, i.e., it assigns operators to operators.

On the other hand, various kinds of pseudodifferential calculi has been developed in mathematics and applied to the study of PDE, geometry and mathematical physics. The main idea behind these theories of pseudodifferential calculi is to construct systematic ways of assigning linear operators to symbol functions, which enables us to translate properties of functions to properties of linear operators.

In this talk, I will introduce a novel pseudodifferential calculus of super operators in the magnetic setting and explain how the Liouville super operator  $L_H$  can be incorporated into this new pseudodifferential theory. Furthermore, the  $L_2$ -boundedness of pseudodifferential super operators will be discussed. Based on the joint work with M. Lein.

Fang Li

Zhejiang University, Hangzhou, China

TITLE: **On 2nd-stage quantization of quantum cluster algebras**

Motivated by the phenomenon that compatible Poisson structures on a cluster algebra play a key role on its quantization (that is, quantum cluster algebra), we introduce the 2nd-stage quantization of a quantum cluster algebra, which means the correspondence between compatible Poisson structures of the quantum cluster algebra and its 2nd-stage quantized cluster algebras. Based on this observation, we find that a quantum cluster algebra possesses a mutually alternating quantum cluster algebra such that their 2nd-stage quantization can be essentially the same.

As an example, we give the 2nd-stage quantized cluster algebra  $A_{p,q}(SL(2))$  of  $Fun_{\mathbb{C}}(SL_q(2))$  and show that it is a non-trivial 2nd-stage quantization, which may be realized as a parallel supplement to two parameters quantization of the general quantum group. As another example, we present a class of quantum cluster algebras with coefficients which possess a non-trivial 2nd-stage quantization. In particular we obtain a class of quantum cluster algebras from surfaces with coefficients which possess non-trivial 2nd-stage quantization.

Finally, we prove that the compatible Poisson structures of a quantum cluster algebra without coefficients is always a locally standard Poisson structure. Following this, it is shown that the 2nd-stage quantization of a quantum cluster algebra without coefficients is in fact trivial.

Si Li

Tsinghua University, Beijing, China

**TITLE: Quantization and index theory**

Abstract: We discuss some basic ideas and various recent mathematical developments on the connection between quantization that arises from topological/chiral field theory and index type theorem. We illustrate renormalization group flow as a version of Hochschild-Kostant-Rosenberg theorem. This comparison reveals a connection between elliptic chiral homology of chiral algebras and a chiral analogue of algebraic index theory. This talk is based mainly on a joint developing program with Zhengping Gui.

**Yong Li**

Tsinghua University, Beijing, China

**TITLE: The asymptotic expansion conjecture on Seifert fibered homology sphere via resurgence theory**

We introduce the Gukov-Pei-Putrov-Vafa (GPPV) invariant (or  $\hat{Z}$  invariant) in Chern-Simons theory associated with a Seifert fibered integral homology sphere (SFIHS). As a corollary of Prof. Sauzin's talk, the GPPV invariant is essentially a partial theta series and a quantum modular form. Moreover, Ecalle's alien derivations provide all components of  $SL(2, \mathbb{C})$ -irreps of the fundamental group of the SFIHS. As the complex variable goes to a rational number, the GPPV invariant becomes the Witten-Reshetikhin-Turaev (WRT) invariant. Simultaneously, only  $SU(2)$  components remain in the transseries expansion. This yields a proof of the so-called asymptotic expansion conjecture of the WRT invariant of SFIHS.

This talk is based on the joint work with J. Andersen, L. Han, W. Mistegard, D. Sauzin and S. Sun.

**Jiasheng Lin**

Sorbonne University, Paris, France

**TITLE:  $P(\phi)_2$  measures on compact Riemannian surfaces, Segal gluing and applications**

We outline the classical Nelson construction of the so-called  $P(\phi)_2$  quantum field theory measure on compact Riemannian surfaces (example. the  $\phi_2^4$  measure), and discuss briefly how this works with the Markov property of the GFF, and the (innocent) "Bayes principle" to give a new proof of the Segal gluing axioms for  $P(\phi)_2$ . Then we discuss a nice consequence of the Perron-Frobenius property of the Segal transfer operator about asymptotics of zeta-determinants on cyclic cover surfaces.

**Sarah-Jean Meyer**

University of Oxford, England

**TITLE: The FBSDE approach to sine-Gordon up to  $6\pi$** 

I will present a stochastic analysis of the sine-Gordon Euclidean quantum field  $(\cos(\beta\Phi))_2$  on the full space up to the second threshold, i.e. for  $\beta^2 < 6\pi$ . The basis of the method is a stochastic quantisation equation given by a forward-backward stochastic differential equation (FBSDE) for a decomposition  $(X_t)_{(t \geq 0)}$  of the interacting Euclidean field  $X_\infty$  along a scale parameter  $t \geq 0$  using an approximate version of the renormalisation flow equation. The FBSDE produces a scale-by-scale coupling of the interacting field with the Gaussian free field without cut-offs and describes the optimiser of a stochastic control problem for the Euclidean QFT. I will explain the general set-up for the FBSDE approach and provide some applications of the FBSDE in the case of the sine-Gordon model to illustrate that it can be used effectively to obtain results about large deviations, integrability, decay of correlations for local observables, singularity with respect to the free field, Osterwalder-Schrader axioms and other properties. This is joint work with Massimiliano Gubinelli.

Martin Peev

Imperial College London, England

TITLE: **Renormalising noncommutative SPDEs**

When solving singular (bounded) operator-valued SPDE's describing the stochastic quantisation of Fermions, unbounded operators naturally appear. However, these cannot be directly equipped with a topology compatible with a fixed-point argument.

I will give a short introduction to Stochastic Quantisation as well as the non-commutative probability necessary to describe Fermions. Then, I will discuss a novel construction, based on ideas from non-commutative geometry, allowing one to find non-commutative "points" in an extension of the algebra of operators. Using these, one can then construct local solutions analogous to pathwise solutions in stochastic PDE's.

This talk is based on joint work with Ajay Chandra and Martin Hairer.

Raphaël Ponge

Sichuan University, Chengdu

TITLE: **Noncommutative geometry and semiclassical analysis**

Semiclassical analysis and noncommutative geometry are distinct fields within the wider area of quantum theory. Bridges between them have been emerging recently. This lays down on operator ideal techniques that are used in both fields. In this talk we shall present semiclassical Weyl's laws for Schrödinger operators on noncommutative manifolds (i.e., spectral triples). This shows that well known semiclassical Weyl's laws in the commutative setting ultimately holds in a purely noncommutative setting. This extends and simplifies previous work of McDonald-Sukochev-Zanin. In particular, this allows us to get semiclassical Weyl's laws on noncommutative tori of any dimension  $n \geq 2$ , which were only accessible in dimension  $n \geq 3$  by the MSZ approach. There are numerous other examples as well. The approach relies on spectral asymptotics for some weak Schatten class operators. As a further application of these asymptotics we obtain far reaching extensions of Connes' integration formulas for noncommutative manifolds. (For Riemannian closed manifolds, Connes' integration shows that Connes' NC integral recaptures the Riemannian measure.)

Alberto Richtsfeld

Universität Potsdam, Germany

TITLE: **Local index theory for the Rarita-Schwinger operator**

We prove the local index theorem for the Rarita-Schwinger operator and higher Dirac operators using Gilkey's invariant theory. That is, we show that the supertrace of the heat kernel of a given geometric operator converges as time goes to zero, and identifies the limit as the Chern-Weil form of the Atiyah-Singer integrand.

David Sauzin

Capital Normal University, Beijing, China on leave from CNRS { IMCCE, Paris, France

TITLE: **Resurgence illustrated on partial theta series**

This will be an initiation to Ecalle's Resurgence theory and Alien Calculus illustrated on the example of partial theta series, based on <https://arxiv.org/abs/2112.15223> and <https://arxiv.org/abs/2310.15029>. What we call partial theta series is a function of the form

$$\Theta(\tau) = \sum_{n>0} f(n)e^{i\pi n^2\tau}.$$

with  $f : \mathbb{Z} \rightarrow \mathbb{C}$  a periodic function, or the product of a power of  $n$  by such a function. The function  $\Theta$  gives rise to divergent asymptotic series at every rational point of the boundary of its domain of definition  $\text{Im}\tau > 0$ . The summability and resurgence properties of these series can be discussed by means of explicit formulas for their formal Borel transforms, as well as the consequences for their modularity properties, or "quantum modularity" in the sense of Zagier's recent theory. Ecalle's "alien derivations" allow one to encode this phenomenon in a kind of "Bridge equation". Interesting examples stem from the study of Gukov-Pei-Putrov-Vafa invariants in Chern-Simons theory, which are the topic of another talk in this workshop.

### Yunhe Sheng

Jilin University, China

**TITLE: Rota-Baxter operators on groups and post-groups**

Rota-Baxter operators on Lie algebras were first studied by Belavin, Drinfeld and Semenov-Tian-Shansky as operator forms of the classical Yang-Baxter equation. Integrating the Rota-Baxter operators on Lie algebras, we introduce the notion of Rota-Baxter operators on Lie groups and more generally on groups. We discuss the notion of post-groups, which is the underlying structure of Rota-Baxter operators on groups. Differentiating post-Lie groups gives rise to post-Lie algebras. Post-groups are also related to braces and Lie-Butcher groups, and give rise to solutions of Yang-Baxter equations. The talk is based on the joint work with Chengming Bai, Li Guo, Honglei Lang and Rong Tang.

### Harprit Singh

Edinburgh University, England

**TITLE: Renormalisation of singular SPDEs on Riemannian manifolds**

I shall discuss a generalisation of the theory of regularity structures which is able to treat singular SPDEs on manifolds with values in vector bundles. After presenting some preliminary background, I shall focus on how positive and negative renormalisation can be implemented in an intrinsic manner. Based on joint work with M. Hairer

### Sumati Surya

Raman Research Institute, Bangalore, India

**TITLE: Quantum dynamics in causal sets**

Causal sets, or locally finite posets, are candidates for a discrete, causal theory of spacetime. In this talk I will review a couple of proposals for constructing the quantum dynamics for causal sets and highlight some of the challenges these bring with them. I will first describe recent results on the quantum partition function over causal sets which show how the discrete Einstein-Hilbert action suppresses an entropically dominant class of non-spacetime-like causal sets, while favouring (at least some) that are like spacetime. I will then describe a more constructive (or fundamentally discrete) approach to causal set dynamics. This is the "sequential growth" paradigm, where the causal set grows element by element. The quantum dynamics of this growth model is given by a decoherence functional, or equivalently, a vector measure. I will discuss recent and ongoing work on a "transfer operator" approach to this dynamics.

## Reiko Toriumi

Okinawa Institute of Science and Technology (OIST) Graduate University, Japan

**TITLE: Renormalisation of enhanced quartic tensor field theories**

Tensor field theory is a quantum field theoretic counterpart of tensor models, which are a random geometric approach to quantum gravity, and can be seen as a higher-dimensional generalisation of matrix model approach to quantum gravity. In tensor models, usually in some large  $N$  limit, graphs called melons dominate and they exhibit branched polymer (tree) like characteristic, undesirable from quantum gravity point of view. One may “enhance” certain interactions so that nonmelonic Feynman graphs contribute to the dominant amplitudes, which consequently may drive us away from branched polymer in the melonic limit. We consider two types of enhanced models  $+$  and  $\times$  with order  $d$  tensor fields  $\phi : (U(1)^D)^d \rightarrow \mathbb{C}$  and with the enhanced quartic interactions of the form  $p^{2a}\phi^4$  reminiscent of derivative couplings expressed in momentum space. Scrutinising the degree of divergence via multiscale renormalisation analysis, we study their renormalisability at all orders of perturbation. We furthermore compute the beta functions of the couplings to understand their renormalisation group flow behavior. At all orders of perturbation, both enhanced models have a constant wave function renormalisation, therefore no anomalous dimension. Nevertheless, both models acquire nontrivial radiative corrections for the coupling constants, exhibiting rather exotic behaviors.

## Yannic Vargas

Technische Universität Graz, Austria

**TITLE: The Steinmann algebra in perturbative quantum field theory and a new basis for Zie**

Based on the work of Bogoliubov-Shirkov, Steinmann, Ruelle, and Epstein-Glaser-Stora, William Norledge introduced an algebraic formalism for perturbative quantum field theory (pQFT) using the theory of Joyal’s species. The approach, which makes use of the notion of Hopf monoid in species developed by Aguiar and Mahajan, allows to study causal perturbation theory using a homomorphism from the Hopf monoid of decorated set compositions (operator-valued distributions decorated with local observables) into the Wick algebra of microcausal polynomial observables.

In this talk we review the construction of the space Zie, a Lie algebra in species closely related to the Steinmann algebra studied in physics. It is generated by the Dynkin elements (introduced in this context by Epstein-Glaser-Stora), modulo some relations known as the Steinmann relations, which are certain four-term linear relations appearing in the foundations of axiomatic quantum field theory. For instance, Steinmann relations have been studied in the context of scattering amplitudes by Caron-Huo et. al., where they appear to be related to cluster algebras. The Steinmann algebra is then an ordinary graded Lie algebra based on the structure map for the adjoint braid arrangement realization of Zie. We present a new combinatorial basis of Zie, based on the notion of infinitesimal bialgebras, and some applications to the study of the Steinmann algebra.

## Hang Wang

East China Normal University, Shanghai, China

**TITLE: Equivariant index theorem on  $R^n$  and continuous fields of  $C^*$ -algebras**

We shall present an equivariant index theorem on Euclidean spaces using a continuous field of  $C^*$ -algebras, which can be described globally as some completion of the convolution algebras of Heisenberg groups. This generalizes the work of Elliott, Natsume and Nest, which is a special case of the algebraic index theorem by Nest-Tsygan. Using our formula, the equivariant index of the Bott-Dirac operators on Euclidean spaces can be explicitly calculated. This is joint work with Baiying Ren and Zijing Wang.

**Xin Wang**

Shandong University, China

TITLE: **Universal structures in Hodge integrals**

In this talk, we will discuss two universal structures in Hodge integrals. One is Virasoro constraints for Hodge integrals of any target varieties. Another one is partial differential equations for higher genus Gromov-Witten invariants from Hodge integrals of any target varieties. This talk is partially based on joint work with Felix Janda.

**Xiaomeng Xu**

Beijing University, China

TITLE: **Yang-Baxter relations, Stokes phenomenon and quantum Riemann-Hilbert-Birkhoff map**

The Stokes phenomenon describes the jump of the asymptotics of solutions of a meromorphic ODE at a pole. We explain how such jump phenomena can be interpreted in terms of various Yang-Baxter type relations. This motivates the introduction of quantum Stokes matrices at a pole of arbitrary order, that gives rise to a quantization of the Riemann-Hilbert-Birkhoff map.

**Jorge Zanelli**

Centro de Estudios Científicos (CECS), Valdivia, Chile

TITLE: **Chern-Simons paradigm for Gravity**

Abstract: Einstein taught us that gravitation -affecting falling apples, orbiting planets, comets and spacecrafts- is a phenomenon described by a dynamical theory for the geometry of spacetime. He also observed that in a sufficiently small neighborhood spacetime resembles Minkowski space, so by a suitable choice of reference frame gravity can be eliminated or faked (Equivalent Principle).

This lecture will review how this construction comes about and why the Chern Simons form plays an exceptional role among all possible gravitation theories for spacetimes of dimensions  $D \neq 4$ . Special attention will be given to the case  $D = 3$ .

**Fabrizio Zanella**

Universität Potsdam, Germany

TITLE: **Conserved currents for the sine-Gordon model, their renormalizability and summability in pAQFT**

We start by presenting the classical theory of the sine-Gordon model. We introduce the so-called Bäcklund transformations and describe how one can use them to obtain an infinite number of conservation laws. Then we formulate the quantum version of the sine-Gordon model in the framework of pAQFT and study the renormalization of the conserved currents. Building on previous results on the S-matrix of the sine-Gordon model, we show that the interacting currents are super-renormalizable and that their associated formal power series are in fact summable.

**Honglian Zhang**

Shanghai University, China

TITLE: **Quantum N-toroidal algebras and extended quantized GIM algebras of N-fold affinization**

We introduce the notion of quantum N-toroidal algebras as natural generalization of the quantum toroidal algebras as well as extended quantized GIM algebras of N-fold affinization. We show that the quantum N-toroidal algebras are quotients of the extended quantized GIM algebras of N-fold affinization, which generalizes a wellknown result of Berman and Moody for Lie algebras. This talk is based on the joint work with Prof. Y. Gao, N. Jing and L. Xia.

### Yuanyuan Zhang

Henan University

TITLE: **Free Rota-Baxter family algebras and free (tri)dendriform family algebras**

Algebraic structures may appear in family version, where the operations are replaced by a family of operations indexed by some set  $\Omega$ , in general endowed with a semigroup structure. Rota-Baxter family algebra comes from the momentum scheme in Quantum Field Theory. We mainly introduce the free Rota-Baxter family algebras and free (tri)dendriform family algebras, then give the relationship between them.

### Guodong Zhou

East China Normal University

TITLE: **Minimal models in algebra and operad theory**

Minimal models of algebras originated from rational homotopy theory and play an important role in algebra. For algebraic structures governed by operads, it is also important to seek minimal models, say, homotopy version of them. For instance, A-infinity algebras are the homotopy version of associative algebras, which can be obtained by Koszul duality. However, for operated algebras such Rota-Baxter algebras or differential algebras with nonzero weights, the minimal models remained open. Recently we found a method to produce minimal models for such operated algebras. This talk is a survey about recent work on this subject.