Local index theory for the Rarita-Schwinger operator

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Algebraic, analytic and geometric structures emerging from quantum field theory
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References

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Classical results

- The Atiyah-Singer index theorem calculates the index of an elliptic operator on a closed manifold in terms of the topology.
- The original proof used methods from K-theory and cobordism theory.
- Over time, new proofs emerged, one of which was the heat kernel method, which led to the local index theorem.

The heat kernel method

- Let $E, F \to M$ be hermitian vector bundles over a closed Riemannian manifold M.
- Let $D : \Gamma(E) \to \Gamma(F)$ be a elliptic differential operator.
- Then D defines two heat semigroups $\exp(-tD^*D)$ and $\exp(-tDD^*)$.
- $\exp(-tD^*D)$ and $\exp(-tDD^*)$ are smoothing operators, i.e. they have smooth Schwartz kernels k_1, k_2. In particular, they are trace-class operators.

The heat kernel method

A simple argument shows that for any $t > 0$

$$\begin{aligned} \text{ind}\, D &= \mathrm{tr}\exp(-tD^*D) - \mathrm{tr}\exp(-tDD^*) \\ &= \int_M (\mathrm{tr}\, k_1(x,x)) - \mathrm{tr}\, k_2(x,x)) dvol_g \\ &:= \int_M (\mathrm{str}\exp(-t\mathcal{D}^2)(x,x)) dvol_g \end{aligned}$$

The **local index theorem** asks the question whether the limit

$$\lim_{t \searrow 0} (\mathrm{str}\exp(-\mathcal{D}^2)(x,x)) dvol_g$$

exists and if so, what the limit is.

The local index theorem for twisted Dirac operators

The local index theorem is true for twisted Dirac operators:

Theorem (Gilkey '73, Atiyah-Bott-Patodi '73, Getzler '83, Bismut '84, Berline-Vergne '85,...)

Let (M, g) be a Riemannian spin manifold and $E \to M$ be a Hermitian vector bundle with connection ∇^E. Then for the twisted Dirac operator D_E, we have

$$\lim_{t \searrow 0} (\mathrm{str}\exp(-tD_E^2)(x,x)) dvol_g = \left(\hat{A}(\nabla^g)(x) \wedge \mathrm{ch}(\nabla^E)(x)\right)_n,$$

where ω_n denotes the n-form part of a mixed-degree differential form ω.

Categories & natural transformations

Let $\mathbf{Man}_n$ be the category of compact, connected, smooth n-manifolds with local diffeomorphisms as morphisms.

- Set
$$\mathrm{Met} : \mathbf{Man}_n^{op} \to \mathbf{Set}$$
to be the functor sending M to the set of metrics $\mathrm{Met}(M)$ on M.
- Set
$$\Omega^q : \mathbf{Man}_n^{op} \to \mathbf{Set}$$
to be the functor sending M to the set $\Omega^q(M)$ of differential q-forms on M.

Riemannian invariants

By a monomial in the partial derivatives of a metric g on $\mathbb{R}^n$, we mean expressions of the form

$$m_\alpha(g) = \partial_x^{\alpha_1} g_{i_1 j_1} \cdot \dots \cdot \partial_x^{\alpha_n} g_{i_n j_n}.$$

A natural transformation $\omega : \mathrm{Met} \to \Omega^q$ is said to be

- *homogeneous of weight* k, if for every $\lambda > 0$ $\omega(\lambda^2 g) = \lambda^k \omega(g)$ holds.
- *regular*, if in coordinates, $\omega(g)$ takes the form

$$\omega(g)(x) = \sum_I \sum_\alpha^{\text{finite}} a_{\alpha,I}(g(x)) \cdot m_\alpha(g)(x) \cdot dx^{i_1} \wedge \dots \wedge dx^{i_q},$$

where $a_{\alpha,I} : Sym_{>0} \to \mathbb{C}$ are C^∞-functions.

Pontryagin forms

- Let Ω_g be the curvature 2-form of the Levi-Civita connection of $g \in \mathrm{Met}(M)$.
- $\mathrm{Pont}(g) = \{P(\Omega_g) | P : \mathfrak{o}(n) \to \mathbb{C}\ O(n) - \text{inv. polynomial}\}$
- The Pontryagin forms are given by

$$\det\left(t + \frac{\Omega_g}{2\pi}\right) = \sum t^{n-2k} p_k(g).$$

- These are generators of $\mathrm{Pont}(g)$.
- $p_k : \mathrm{Met} \to \Omega^{4k}$ defines a regular, homogeneous natural transformation of weight 0.

Gilkey's Theorem

Theorem (Gilkey '73, Atiyah-Bott-Patodi '73)

The only regular, homogeneous natural transformations $\omega : \mathrm{Met} \to \Omega^q$ of weight ≥ 0 have values in the ring $\mathrm{Pont}(g)$ generated by the Pontryagin forms of g, and these have weight 0.

Geometric structures

- Let $G_n \in \{\mathrm{O}(n), \mathrm{SO}(n), \mathrm{Pin}(n), \mathrm{Spin}(n)\}$.
- G_n has a natural action on $\mathbb{R}^n$.
- Given a manifold M^n, there are topological conditions for G_n-structures, the parameter spaces for the different G_n-structures are again determined by the topology of M.
- There is a subcategory $G_n - \mathbf{Man}_n$ of $\mathbf{Man}_n$, whose objects are the manifolds admitting G_n-structures.
- There is a functor

$$G_n - \mathrm{Str} : G_n - \mathbf{Man}_n^{op} \to \mathbf{Set},$$

sending a manifold M to the set of different G_n-structures on M.

Geometric structures

The set of orientations $\mathcal{O}(M)$ on M is given by $\pi_0(\tilde{M})$, where $\tilde{M}$ is the orientation double cover of M. $\mathcal{O}$ is a functor from $\mathbf{Man}_n^{op}$ to **Set**.

G_n	$\mathrm{Obj}(G_n - \mathbf{Man}_n)$	$G_n - \mathrm{Str}$
$\mathrm{O}(n)$	$\mathrm{Obj}(\mathrm{Man}_n)$	$*$
$\mathrm{SO}(n)$	M with $w_1(M) = 0$	$\mathcal{O}$
$\mathrm{Pin}(n)$	M with $w_1(M) = w_2(M) = 0$	$H^1(\cdot, \mathbb{Z}_2)$
$\mathrm{Spin}(n)$	M with $w_1(M) = w_2(M) = 0$	$\mathcal{O} \times H^1(\cdot, \mathbb{Z}_2)$

A G_n-manifold (M, α) is a pair with $M \in \mathrm{Obj}(G_n - \mathbf{Man_n})$ and $\alpha \in G_n - \mathrm{Str}(M)$.

Constructions emerging from geometric structures

Let $M \in \mathrm{Obj}(G_n - \mathbf{Man}_n)$.

- Each $g \in \mathrm{Met}(M)$ and $\alpha \in G_n - \mathrm{Str}(M)$ determines a G_n-principal bundle $PG_{g,\alpha}(M)$.
- The metric g induces a Levi-Civita connection 1-form ω^{LC} on $PG_{g,\alpha}(M)$.
- A G_n-representation $\rho : G_n \to \mathrm{End}(V)$ induces an associated vector bundle $E_{V,g,\alpha} = PG_{g,\alpha}(M) \times_\rho V$.
- The connection 1-form ω^{LC} induces a covariant dericative ∇^{LC} on $E_{V,g,\alpha}$.

Geometric operators

Definition

A geometric symbol σ is a G_n-equivariant map

$$\sigma : \mathbb{R}^n \to \mathrm{Hom}(V, W),$$

V, W are hermitian representations of G_n. For a Riemannian G_n-manifold (M, g, α), σ defines an (elliptic) first-order differential operator

$$D_{\sigma,g,\alpha} := \bar{\sigma} \circ \nabla^{LC} : C^\infty(M, E_{V,g,\alpha}) \to C^\infty(M, E_{W,g,\alpha}),$$

where $\bar{\sigma}$ is the to σ associated section of $T^*M \otimes \mathrm{Hom}(E_{V,g,\alpha}, E_{W,g,\alpha})$. Operators constructed in this way are called geometric.

Geometric operators: Properties

- For $\lambda > 0$ there exists canonical vector bundle isomorphisms $\epsilon_V : E_{V,g,\alpha} \to E_{V,\lambda^2 g,\alpha}$, $\epsilon_W : E_{W,g,\alpha} \to E_{W,\lambda^2 g,\alpha}$ such that

$$\lambda D_{\sigma,\lambda^2 g,\alpha} \circ \epsilon_V = \epsilon_W \circ D_{\sigma,g,\alpha}.$$

- In coordinates, the coefficients of $D_{\sigma,g\alpha}$ are of the form

$$a(x,g) = \sum_{\alpha} a_\alpha(g(x)) m_\alpha(g)(x),$$

where $a_\alpha : Sym_{>0}(n) \to \mathbb{C}$ are C^∞-functions.

Definition of Chiral Geometric Symbol

Definition

Let $H_n \in \{\mathrm{Pin}(n), \mathrm{O}(n)\}$, V be an H_n-representation, and $G_n \subseteq H_n$ be the connected component of $1 \in H_n$. A chiral (G_n-)geometric symbol (σ, ε) consists of:

- A H_n-geometric symbol $\sigma : \mathbb{R}^n \to \mathrm{Hom}(V)$,
- a H_n-geometric map $\varepsilon : \Lambda^n\mathbb{R}^n \to \mathrm{Hom}(V)$,

such that

- $\sigma(\xi)$ is skew-adjoint for all $\xi \in \mathbb{R}^n$
- $\varepsilon(\mathbf{e}_1 \wedge ... \wedge \mathbf{e}_n)^2 = 1$
- $\sigma \cdot \varepsilon(\mathbf{e}_1 \wedge \cdots \wedge \mathbf{e}_n) = -\varepsilon(\mathbf{e}_1 \wedge \cdots \wedge \mathbf{e}_n) \cdot \sigma.$

Geometric Operators from Chiral Symbols

- Let $G_n \in \{\mathrm{SO}(n), \mathrm{Spin}(n)\}$ and (σ, ε) be an G_n-geometric symbol.
- $V^{\pm}$ are the ± 1-eigenspaces of $\varepsilon(\mathbf{e}_1 \wedge \cdots \wedge \mathbf{e}_n)$.
- $V^{\pm}$ are G_n-representations and

$$\sigma^{\pm} : \mathbb{R}^n \otimes V^{\pm} \to V^{\mp}$$

are G_n-equivariant.

Geometric Operators from Chiral Symbols

- Given a Riemannian G_n-manifold (M, g, α), the chiral geometric symbol induces a $\mathbb{Z}_2$-grading:

$$E_V = E_{V^+} \oplus E_{V^-}$$

- $E_{V^\pm}$ can be identified as the ± 1-eigenspaces of $\bar{\varepsilon}(dvol_g)$.
- Geometric operators obtained:

$$D_\sigma : E_V \to E_V, \quad D_{\sigma^+} : E_{V^+} \to E_{V^-}, \quad D_{\sigma^-} : E_{V^-} \to E_{V_+}.$$

- Such that:

$$D_\sigma = \begin{pmatrix} 0 & D_{\sigma^-} \\ D_{\sigma^+} & 0 \end{pmatrix}$$

- Since D_σ is self-adjoint, we have:

$$(D_{\sigma^+})^* = D_{\sigma^-}.$$

Proposition

Let $G_n \in \{\mathrm{SO}(n), \mathrm{Spin}(n)\}$ *and* (σ, ε) *be a chiral geometric symbol. Let* (M, g, α) *be a* G_n*-manifold and* $\bar{\alpha} \in G_n - \mathrm{Str}(M)$ *be the* G_n*-structure that is obtained from* α *by reversing the orientation. Then we have the followng equalities*

$$E_{V,\alpha} = E_{V,\bar{\alpha}}, \quad E_{V^{\pm},\alpha} = E_{V^{\mp},\bar{\alpha}}, \quad D_{\sigma^{\pm},\alpha} = D_{\sigma^{\mp},\bar{\alpha}}.$$

Higher Dirac operators

- For $n = 2k$, let $V_j^{\pm}$ be the irreducible representation of $\mathrm{Spin}(n)$ with dominant weight

$$\lambda_j^{\pm} = \Big(\underbrace{\frac{3}{2}, \dots, \frac{3}{2}}_{j \text{ times}}, \frac{1}{2}, \dots, \frac{1}{2}, \pm\frac{1}{2}\Big).$$

- $V_j = V_j^+ \oplus V_j^-$ is a $\mathrm{Pin}(n)$-representation.
- V_j appears once in $\Sigma_n \otimes \Lambda^j \mathbb{C}^n$:

$$\Sigma_n \otimes \Lambda^j \mathbb{C}^n = V_j \oplus W, \quad \Sigma_n^{\pm} \otimes \Lambda^j \mathbb{C}^n = V_j^{\pm} \oplus W_{\pm}.$$

- All sums are $\mathrm{Spin}(n)$-equivariant, the first is $\mathrm{Pin}(n)$-equivariant.
- The (orthogonal) projection $\pi_j : \Sigma_n \otimes \Lambda^j \mathbb{C}^n \to V_j$ is $\mathrm{Pin}(n)$-equivariant.

Higher Dirac operators

- Define twisted Clifford multiplication γ_j and involution $\omega_{\mathbb{C}} \otimes \mathrm{id}_{\Lambda^j \mathbb{C}^n}$,

$$\omega_{\mathbb{C}} = i^k e_1 \cdots e_n.$$

- Set $\sigma_j = \pi_j \circ \gamma_j|_{\mathbb{R}^n \otimes V_j}$, then $(\sigma_j, \omega_{\mathbb{C}} \otimes \mathrm{id}|_{V_j})$ defines a chiral geometric symbol.
- Operators D_j obtained are called higher Dirac operators.
- D_0 is the Dirac operator, D_1 is the Rarita-Schwinger operator.
- ind $D_{j,+} = \langle (\mathrm{ch}(\Lambda^j T^*_{\mathbb{C}} M) + \mathrm{ch}(\Lambda^{j-1} T^*_{\mathbb{C}} M)) \hat{\mathrm{A}}(M), [M] \rangle$

Index and heat kernel

Let (σ, ε) be a chiral geometric symbol, (M, g, α) closed and connected Riemannian G_n-manifold. Let

$$D : \Gamma(E) \to \Gamma(E)$$

the geometric symbol obtained from σ, $D_\pm : \Gamma(E_\pm) \to \Gamma(E_\mp)$ the chiral parts. $\exp(-tD^2)$ is a smoothing operator, w.r.to the splitting $E = E_+ \oplus E_-$:

$$\exp(-tD^2) = \begin{pmatrix} \exp(-tD_- D_+) & 0 \\ 0 & \exp(-tD_+ D_-) \end{pmatrix}.$$

$$\text{ind } D_+ = \int^{\mathcal{O}} \mathrm{str}(\exp(-tD^2)(x,x)) dvol_{g,\mathcal{O}}$$

$\mathcal{O}$ is the orientation induced from α,

$$\mathrm{str}(A) = \mathrm{tr}(\bar{\varepsilon}(dvol_{g,\mathcal{O}})A), \quad A \in \mathrm{Hom}(E,E).$$

Construction of asymptotic expansion

Obtain an asymptotic expansion

$$\exp(-tD^2)(x,x) \sim \sum \Phi_k(x) t^{\frac{k-n}{2}} :$$

Let $\sigma_{D^2} = \sum_{k\leq 2} a_k$ is the total symbol of D^2 in coordinates, a_k being the homogeneous parts of degree k. Approximate the symbol of the parametrix $(D^2-\lambda)^{-1}$ by inverting the symbol of $D^2-\lambda$ formally:

- $b_0(x,\xi.\lambda) = (a_2(x,\xi)-\lambda)^{-1}$,
- For $D_x^\alpha = (-i)^{|\alpha|}\partial_x^\alpha$,

$$b_k = -\left(\sum_{\substack{|\alpha|+j+l=k \\ j<k}} \frac{1}{\alpha!} \frac{\partial^\alpha b_j}{\partial \xi^\alpha} \cdot D_x^\alpha a_{2-l} \right) \cdot b_0$$

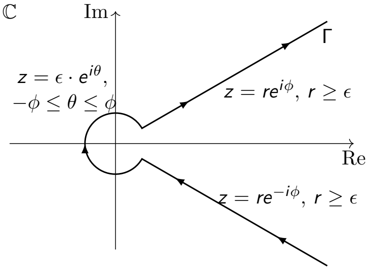
.

The heat coefficients

The asymptotic expansion of the heat kernel is given by

$$\Phi_k(x) = \sqrt{\det g(x)}^{-1} \frac{1}{2\pi i} \int \int_\Gamma e^{-\lambda} b_k(x, \xi, \lambda) d\lambda d\xi,$$

where Γ is given by



The heat coefficents

- Let Φ_k be the asymptotic expansion of $\exp(-tD^2_{\sigma,g,\alpha})$.
- Set $\omega_k^{(\sigma,\varepsilon)}(M,g,\alpha)(x) = \mathrm{str}(\Phi_k(x))dvol_{g,\mathcal{O}}$.
- ind $D^+_{\sigma,g,\alpha} = \int_M^{\mathcal{O}} \omega_n^{(\sigma,\varepsilon)}$.

Heat coefficients as natural transformations

$\omega_k^{(\sigma,\varepsilon)}(M, g, \alpha)$ does not depend on α:

- $\omega_k^{(\sigma,\varepsilon)}(M, g, \alpha)$ is a local construction.
- Locally, a G_n-structure is determined by the orientation.
- $\omega_k^{(\sigma,\varepsilon)}(M, g, \alpha)(x) = \mathrm{tr}(\bar{\varepsilon}(dvol_{g,\mathcal{O}})\Phi_k(x))dvol_{g,\mathcal{O}}$
- For the reversed orientation $\bar{\mathcal{O}}$ we have $dvol_{g,\bar{\mathcal{O}}} = -dvol_{g,\mathcal{O}}$.

Proposition

The kth heat coefficient $\omega_k^{(\sigma,\varepsilon)}$ defines a natural transformation $\mathrm{Met} \to \Omega^n$.

Regularity of the heat coefficients

Lemma

The natural transformation $\omega_k^{(\sigma,\varepsilon)}$ is regular.

Proof of regularity

Sketch of proof:

- In coordinates, the coefficients of $D_{\sigma,g}$ are of the form

$$a(x,g) = \sum_{\alpha} a_\alpha(g(x)) m_\alpha(g)(x), \qquad (*)$$

  where $a_\alpha : Sym_{>0}(n) \to \mathbb{C}$ are C^∞-functions.
- Functions of the form $*$ are closed under addition, multiplications and taking derivatives in x.
- By carefully going through the construction of $\omega^{\sigma,\varepsilon}$ one obtains regularity.

Homogeneity of the heat coefficients

Lemma

The natural transformation $\omega_k^{(\sigma,\varepsilon)}$ *is homogeneneous of weight* $\frac{n-k}{2}$ *in* g*, i.e.*

$$\omega_k(\lambda^2 g) = \lambda^{n-k}\omega_k(g).$$

This follows from $D_{\sigma,\lambda^2 g,\alpha} \cong \frac{1}{\lambda} D_{\sigma,g,\alpha}$.

Preliminary local index theorem

Theorem

For $k < n$, the heat coefficient $\omega_k^{(\sigma,\varepsilon)}$ is zero, and $\omega_n^{(\sigma,\varepsilon)} \in \mathrm{Pont}(g)$. In particular, if (M, g, α) is a closed G_n-manifold and $D_{g,\alpha}$ the induced geometric operator, $\mathrm{str}\left(e^{-tD_{g,\alpha}^2}(x,x)\right) dvol_g$ converges for $t \searrow 0$ with

$$\lim_{t \searrow 0} \mathrm{str}\left(e^{-tD_{g,\alpha}^2}(x,x)\right) dvol_g = \omega_n(g)(x).$$

Atiyah-Singer index theorem

On an oriented manifold M^n with orientation $\mathcal{O}$, denote by $\chi(TM_{\mathcal{O}}) \in H^n(M, \mathbb{R})$ the Euler class of the TM with respect to the orientation $\mathcal{O}$.

Theorem (Atiyah-Singer)

Let (σ, ε) be a chiral G_n-geometric symbol for $n = 2m$ even and (M, g, α) be a Riemannian G_n-manifold. Then the characteristic class $(\mathrm{ch}(E_{+,g,\alpha}) - \mathrm{ch}(E_{-,g,\alpha}))/\chi(TM_{\mathcal{O}}) \in H^*(M, \mathbb{R})$ is well-defined and

$$\mathrm{ind}(D_{+,g,\alpha}) = (-1)^m \left(\frac{\mathrm{ch}(E_{+,g,\alpha}) - \mathrm{ch}(E_{-,g,\alpha})}{\chi(TM_{\mathcal{O}})} \cdot \hat{A}(M)^2 \right) [M_{\mathcal{O}}].$$

Atiyah-Singer integrand

- Let V be the G_n-module σ acts on.
- Let $\tilde{E}_+$, $\tilde{E}_-$, $\tilde{T}$ be associated bundles to the G_n-representations V_+, V_-, $\mathbb{R}^n$ on the classifying space BG_n.
- The cohomology class

$$\frac{\mathrm{ch}(\tilde{E}_+) - \mathrm{ch}(\tilde{E}_-)}{\chi(\tilde{T})}\hat{A}(\tilde{T})^2 \in H^*(BG_n, \mathbb{R})$$

  is well-defined.
- By Chern-Weil theory, there exists a G_n-invariant polynomial $P_{\sigma,\varepsilon} : \mathfrak{g}_n \to \mathbb{R}$ representing the above cohomology class.

- The mixed-degree form $P_{(\sigma,\varepsilon)}(\bar{\Omega}^{LC}_{g,\alpha})$ does not depend on α.
- $\omega_{(\sigma,\varepsilon)} : g \mapsto (P_{(\sigma,\varepsilon)}(\bar{\Omega}^{LC}_{g})_n$ defines a zero-homogeneous, regular natural transformation $\mathrm{Met} \to \Omega^n$.
- For all G_n-manifolds (M, g, α),

$$\int_M \omega_n = \mathrm{ind}(D_{+,g,\alpha}) = \int_M \omega_{(\sigma,\varepsilon)}.$$

Thom's Theorem

Applying Gilkey's Theorem,

$$\omega_n = \sum_I a_I p_I, \quad \omega_{(\sigma,\varepsilon)} = \sum_I b_I p_I \quad a_I, b_I \in \mathbb{C},$$

where I runs over all partitions of n and $p_{(i_1,\dots,i_r)} = p_{i_1} \wedge \cdots \wedge p_{i_r}$, p_i denotes the i-th Pontryagin form. To deduce $a_I = b_I$, we use the following Theorem by Thom:

Theorem

Let M_1 be the $K3$-surface and $M_i = \mathbb{H}P^i$ for $i \geq 2$. Then M_i is spinnable and the matrix

$$(p_I(M_{j_1} \times \cdots \times M_{j_k}))_{I,J \text{ partitions of } k}$$

is non-singular.

The local index theorem

Rewrite

$$\frac{\mathrm{ch}(\tilde{E}_+) - \mathrm{ch}(\tilde{E}_-)}{\chi(\tilde{T})}(\nabla^{LC,g}) \cdot \hat{\mathrm{A}}(\nabla^{LC,g})^2 = P_{(\sigma,\varepsilon)}(\bar{\Omega}_g^{LC}).$$

Our discussion shows:

Theorem

Let (σ, ε) be a chiral G_n-geometric symbol for $n = 2m$ even. Let (M, g, α) be a Riemannian G_n-manifold and $D_g \colon C^\infty(M, E) \to C^\infty(M, E)$ be the induced geometric operator. Then the equality

$$\lim_{t \searrow 0} \mathrm{str}\left(e^{-tD_g^2}(x,x)\right) dvol_g =$$
$$= (-1)^m \left(\frac{\mathrm{ch}(\tilde{E}_+) - \mathrm{ch}(\tilde{E}_-)}{\chi(\tilde{T})} (\nabla^{LC,g})(x) \cdot \hat{\mathrm{A}}(\nabla^{LC,g})(x)^2 \right)_n$$

holds.

Local index theorem for the Rarita-Schwinger operator

Corollary

Let $Q = D_1$ *be the Rarita-Schwinger operator on an even-dimensional Riemannian spin-manifold* (M, g)*. Then*

$$\lim_{t \searrow 0} \mathrm{str}\left(e^{-tQ^2}(x,x)\right) dvol_g =$$

$$= \left(\hat{\mathrm{A}}(\nabla^{LC,g})(x)\left(\mathrm{ch}(\tilde{T}_{\mathbb{C}})(\nabla^{LC,g})(x) + 1\right)\right)_n .$$

Corollary

Let D_j be the higher Dirac operator on an even dimensional Riemannian spin-manifold (M,g). Then

$$\lim_{t\searrow 0} \operatorname{str}\left(e^{-tD_j^2}(x,x)\right) dvol_g =$$

$$= \left(\hat{\mathrm{A}}(\nabla^{LC,g})(x)\left(\mathrm{ch}(\Lambda^j \tilde{T}_{\mathbb{C}})(\nabla^{LC,g})(x) + \mathrm{ch}(\Lambda^{j-1}\tilde{T}_{\mathbb{C}})(\nabla^{LC,g})(x)\right)\right)_n .$$

Thank you for your attention!

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